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Surface-induced scattering in resistivity of thin conductive films

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1 Surface-induced scattering in resistivity of thin conductive films. Theory of perturbations

1.1 Calculation in the framework of the "first" approach

In the present work the dependence of the electrical conductivity σ on the thickness d of ultrathin conductive films is investigated. Particularly, one studies the contribution to the resistivity caused by elastic interaction of the carriers with roughness of the film's surface. Number of studies had shown that the resistivity is determined essentially by surface scattering for thickness up to hundreds Å˙
For ideally smooth film of thickness d, lying along the plane OXY, surfaces are determined by expressions $z = \pm\frac{1}{2}d$. In this case Hamiltonian of a charge carrier is written as

$$H_0 = -\frac{\hbar^2}{2m}\Delta + V\theta(z - \frac{1}{2}d) + V\theta(-z - \frac{1}{2}d), \quad (1)$$

where $\theta(z)$ - is Heaviside step function, and V - is height of the potential well, meaning the "work function", m - is the effective mass of the charge carrier.
Eigenstates and eigenvalues H_0 are

$$\langle \mathbf{r}|\nu\mathbf{k}\rangle = S^{-1/2}e^{i\mathbf{k}\cdot\boldsymbol{\rho}}\phi_\nu(z), E_{\nu\mathbf{k}} = E_\nu + \frac{\hbar^2k^2}{2m}, \quad (2)$$

where S - is area of the film surface, ν - is the number of the subband, and $\boldsymbol{\rho}$ and $\mathbf{k}$ - are 2D vectors in real and reciprocal spaces, respectively. If we assume that one of the surfaces is not ideal, e.g. the surface near $z = \frac{1}{2}d$, then its equation becames $z = \frac{1}{2}d + f(\boldsymbol{\rho})$, where adopted $f(\boldsymbol{\rho}) \ll d$. It would be possible to take into account roughness of both surfaces, but in this case it does not change the overall picture, and simplifies calculations. Now the Hamiltonian of the system turns to $H_0 + U$, where

$$U = V\big[\theta\big(z - \frac{1}{2}d - f(\boldsymbol{\rho})\big) - \theta(z - \frac{1}{2}d)\big] \approx -Vf(\boldsymbol{\rho})\delta(z - \frac{1}{2}d) \quad (3)$$

We will carry out the calculations in the lowest order of roughness $f(\rho)$. The expression for conductivity of 2D degenerate gas, whose carriers are elastically scattered by potential U is given (see [2]) in the following form

$$\sigma = \frac{Sm^2e^2}{\pi^2\hbar^6 d}\sum_{\nu=1}^{N}\sum_{\nu'=1}^{N}(E_F - E_\nu)(E_F - E_{\nu'})[C^{-1}(E_F)]_{\nu\nu'}, \quad (4)$$

where E_F is Fermi energy and N is the number of filled levels, i.e. the number of subbands that have E_ν less than E_F. $[C^{-1}(E)]_{\nu\nu'}$ is the inverse matrix to $[C(E)]_{\nu\nu'}$, determined by the elements

$$[C(E)]_{\nu\nu'} = \frac{2\pi}{\hbar} \sum_{\mathbf{k}} \sum_{\mathbf{k}'} \Big\{ \delta_{\nu,\nu'} \sum_{\mu} |\langle \nu\mathbf{k}|U|\mu\mathbf{k}'\rangle|^2 k^2 \delta(E - E_{\nu\mathbf{k}})\delta(E - E_{\mu\mathbf{k}'})$$

$$-|\langle \nu\mathbf{k}|U|\nu'\mathbf{k}'\rangle|^2 \mathbf{k}\cdot\mathbf{k}' \delta(E - E_{\nu\mathbf{k}})\delta(E - E_{\nu'\mathbf{k}'}) \Big\} \quad (5)$$

When U is given by Eq. (3), it yields

$$S|\langle \nu\mathbf{k}|U|\nu'\mathbf{k}'\rangle|^2 = A_\nu A_{\nu'} \int_S d\boldsymbol{\rho} e^{i(\mathbf{k}'-\mathbf{k})\cdot\boldsymbol{\rho}} \langle f(\boldsymbol{\rho}')f(\boldsymbol{\rho}'+\boldsymbol{\rho})\rangle. \quad (6)$$

Here we denote $A_\nu = V\phi_\nu^2(\frac{1}{2}d)$ and autocorrelation function

$$\langle f(\boldsymbol{\rho}')f(\boldsymbol{\rho}'+\boldsymbol{\rho})\rangle = \frac{1}{S}\int_S d\boldsymbol{\rho}' f(\boldsymbol{\rho}')f(\boldsymbol{\rho}+\boldsymbol{\rho}'), \quad (7)$$

which characterizes roughness of the surface. If Δ determines mean square height "hills" of the surface, and ξ is a characteristic correlation length of roughness, then the autocorrelation function of an isotropic surface can be written as

$$\langle f(\rho')f(\boldsymbol{\rho}'+\boldsymbol{\rho})\rangle = \Delta^2 G(\rho/\xi), \quad (8)$$

By defining $F(q)$ as Fourier image of $G(\rho)$, one gets

$$S|\langle \nu\mathbf{k}|U|\nu'\mathbf{k}'\rangle|^2 = A_\nu A_{\nu'} \Delta^2 \xi^2 F(\xi|\mathbf{k}-\mathbf{k}'|). \quad (9)$$

Then conducting integration in Eq. (5), it comes to

$$[C(E)]_{\nu\nu'} = \frac{S\Delta^2\xi^2 m^2}{4\pi^2\hbar^5} A_\nu \int_0^{2\pi} d\theta \left[\delta_{\nu,\nu'} k_\nu^2 \sum_{\mu=1}^{N} A_\mu F(\xi k_{\nu\mu}) - A_{\nu'} k_\nu k_{\nu'} \cos\theta F(\xi k_{\nu\nu'}) \right], \quad (10)$$

where $k_\nu = [(2m/\hbar)(E_F - E_\nu)]^{1/2}$ (Fermi wave vector), $k_{\nu\nu'} = (k_\nu^2 + k_{\nu'}^2 - 2k_\nu k_{\nu'} \cos\theta)^{1/2}$.

The calculation can be simplified in the limiting case, when the correlation length ξ is much less than k_1^{-1}, where k_1 is the greatest Fermi wave vector k_ν. In the limit $\xi k_1 \ll 1$, we have $\xi k_{\nu\nu'} \ll 1$ for all $\nu, \nu' \leq N$ and

$$[C(E)]_{\nu\nu'} = \delta_{\nu,\nu'} \frac{S\Delta^2\xi^2 m^2}{2\pi\hbar^5} F(0) k_\nu^2 A_\nu \sum_{\mu=1}^{N} A_\mu. \quad (11)$$

Furthermore, we will assume the wavefunction of the carrier concentrated inside the film by setting highest potential barrier $V \to \infty$. Then one will prove that

$$\lim_{V\to\infty} A_\nu = \frac{\hbar^2\pi^2\nu^2}{md^3}. \quad (12)$$

Let us consider a rectangular potential well of height V and width d. Now change the width to $d+\Delta d$. Then the potential energy will change correspondingly to

$$V[\theta(z-\frac{1}{2}d-\Delta d)-\theta(z-\frac{1}{2}d)]\approx -V\delta(z-\frac{1}{2}d)\Delta d.$$

According to the theory of perturbations the correction of the first order to energy ν's level is

$$\Delta E_\nu=\langle\nu|-V\delta(z-\frac{1}{2}d)\Delta d|\nu\rangle=-V\phi(\frac{1}{2}d)\Delta d.$$

For the well with infinitely high walls the energy levels are determined by the following expression

$$E_\nu=\frac{\hbar^2\pi^2\nu^2}{2md^2}.$$

From this formula and the expression for the correction Δ_ν one concludes that

$$\frac{\hbar^2\pi^2\nu_2}{md^3}=V\phi^2(\frac{1}{2}d)=A_\nu,$$

and it proves Eq. (12).

Within the before mentioned assumptions the matrix $[C(E)]_{\nu\nu'}$ is diagonal and conductivity is

$$\sigma\approx\frac{e^2}{\hbar}\frac{d^5}{2\pi^6\Delta^2\xi^2}\frac{\pi}{F(0)}\frac{6}{N(N+1)(2N+1)}\sum_{\nu=1}^{N}\frac{k_\nu^2}{\nu^2}. \quad (13)$$

All the reasonings and conclusions made are one ("first") of the methods to calculate conductivity, e.g. exposed in [1].

When going to look into the method detaily, we may state a question about correctness to use "perturbation" U Hamiltonian in the form of δ-function together with limiting transition $V\to\infty$. The replacement of the subtraction of θ-functions by δ-function practically means a replacement of the wavefunction $\phi(z)$ by a constant within the interval of order $f(\boldsymbol{\rho})$. That happens in the calculation of the matrix elements $\langle\nu\mathbf{k}|U|\mu\mathbf{k'}\rangle$.

The wavefunction at $z>\frac{L}{2}$ has the form

$$\phi(z)\sim e^{-[\frac{2m}{\hbar^2}(V-E)]^{1/2}z}=e^{-\varkappa z}$$

then, the conduction of smallness $f(\boldsymbol{\rho})$ must be written as

$$\varkappa f\ll 1 \quad (14)$$

The limiting case $V\to\infty$ is provided by $\varkappa\to\infty$, however $f(\boldsymbol{\rho})$ is assumed to be finite. That obviously leads to the violation of Eq. (14).

1.2 Statement of the "second" approach

Let us try to change the logics of "first" method, so that we do not need the condition (12). Consider a carrier located in a well with infinitely high walls with the corresponding Hamiltonian

$$H_{(z)} = -\frac{\hbar^2}{2m}\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\Big) \tag{15}$$

together with boundary conditions

$$\psi(0) = \psi(d + f(\boldsymbol{\rho})) = 0, \tag{16}$$

where like before $\boldsymbol{\rho} = (x, y)$. If one turns to a "curved" coordinate system (x', y', z') settled by the equations

$$x = x' \tag{17}$$

$$y = y' \tag{18}$$

$$z = z'\big(1 + f(\boldsymbol{\rho}')\big). \tag{19}$$

One should note that here $f(\boldsymbol{\rho}')$ is a relative height of the relief (unitless quantity).
In the new coordinate system the Hamiltonian will have a different form

$$-\frac{\hbar^2}{2m}\Delta_{\boldsymbol{\rho},z} \rightarrow -\frac{\hbar^2}{2m}\Delta_{\boldsymbol{\rho}',z'} - \frac{\hbar^2}{2m}\widetilde{\Delta}, \tag{20}$$

for plain boundary conditions

$$\psi(0) = \psi(d) = 0, \tag{21}$$

where the perturnation is

$$H = -\frac{\hbar^2}{2m}\widetilde{\Delta} = -\frac{\hbar^2}{2m}\Bigg\{\Big[\frac{1}{(1+f)^2} - 1\Big]\frac{\partial^2}{\partial z'^2} - \frac{2}{1+f}\Big(\frac{\partial f}{\partial x'}\frac{\partial}{\partial x'} + \frac{\partial f}{\partial y'}\frac{\partial}{\partial y'}\Big)z'\frac{\partial}{\partial z'} + \\ + \frac{2}{(1+f)^2}\Big[\Big(\frac{\partial f}{\partial x'}\Big)^2 + \Big(\frac{\partial f}{\partial y'}\Big)^2\Big]z'\frac{\partial}{\partial z'} - \frac{1}{1+f}\Big(\frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2}\Big)z'\frac{\partial}{\partial z'} + \\ + \frac{1}{(1+f)^2}\Big[\Big(\frac{\partial f}{\partial x'}\Big)^2 + \Big(\frac{\partial f}{\partial y'}\Big)^2\Big]z'^2\frac{\partial^2}{\partial z'^2}\Bigg\}. \tag{22}$$

Now let us consider only of the first order of perturbation in f. Note that combinations like $z'\frac{\partial}{\partial z'}$ and $z'^2\frac{\partial^2}{\partial z'^2}$ act on the lateral part of the wavefunction $\sin(\frac{\pi\nu}{d}z')$, does not affect f and are of order unity. The resulting Hamiltion of perturbation is

$$H = -\frac{\hbar^2}{2m}\Big\{-2f(\boldsymbol{\rho}')\frac{\partial^2}{\partial z'^2} - \Big(\frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2}\Big)z'\frac{\partial}{\partial z'} - 2\Big(\frac{\partial f}{\partial x'}\frac{\partial}{\partial x'} + \frac{\partial f}{\partial y'}\frac{\partial}{\partial y'}\Big)z'\frac{\partial}{\partial z'}\Big\}. \tag{23}$$

1.3 Perturbation theory for the operator H

The solution of Schrodinger's equation for the unperturbated problem consists of the energetic spectrum:

$$E_\nu = \frac{\pi^2\hbar^2\nu^2}{2md^2}. \tag{24}$$

and eigenfunctions:

$$\langle \mathbf{r}|\nu\mathbf{k}\rangle = \sqrt{\frac{2}{Sd}}\sin(\frac{\pi\nu}{d}z')e^{i\mathbf{k}\cdot\boldsymbol{\rho}}. \tag{25}$$

If Fourier-image of the roughness f is

$$F_f(\mathbf{q}) = \frac{1}{4\pi^2}\int e^{-i\mathbf{q}\cdot\mathbf{p}}f(\boldsymbol{\rho})d\boldsymbol{\rho}, \tag{26}$$

then the autocorrelation function

$$\langle f(\boldsymbol{\rho}')f(\boldsymbol{\rho}'+\boldsymbol{\rho})\rangle = \frac{1}{S}\int_S d\boldsymbol{\rho}' f(\boldsymbol{\rho}')f(\boldsymbol{\rho}+\boldsymbol{\rho}') = \frac{4\pi^2}{S}\int |F_f(\mathbf{q})|^2 e^{i\mathbf{q}\cdot\boldsymbol{\rho}}d\mathbf{q}, \tag{27}$$

and hence,

$$|F_f(\mathbf{q})|^2 = S\int e^{-i\mathbf{q}\cdot\boldsymbol{\rho}}\langle f(\boldsymbol{\rho}')f(\boldsymbol{\rho}'+\boldsymbol{\rho})\rangle d\boldsymbol{\rho}. \tag{28}$$

Calculated diagonal by ν matrix elements of the operator H are

$$\langle \nu\mathbf{k}|H|\nu\mathbf{k}'\rangle = -\frac{\hbar^2}{mS}\int e^{i\boldsymbol{\rho}\cdot(\mathbf{k}'-\mathbf{k})}\Big(\frac{\pi^2\nu^2}{d^2}f + \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + 2i\big(k'_x\frac{\partial f}{\partial x'} + k'_y\frac{\partial f}{\partial y'}\big)\Big)d\boldsymbol{\rho} =$$
$$= \frac{4\pi^2\hbar^2}{mS}\Big[\frac{k^2-k'^2}{4} - \frac{\pi^2\nu^2}{d^2}\Big]F_f(\mathbf{k}-\mathbf{k}'). \tag{29}$$

Non-diagonal elements ($\nu \neq \nu'$) are :

$$\langle \nu\mathbf{k}|H|\nu'\mathbf{k}'\rangle = \frac{4\pi^2\hbar^2}{mS}\frac{(-1)^{\nu+\nu'}\nu'\nu}{(\nu^2-\nu'^2)}(k^2-k'^2)^2F_f(\mathbf{k}-\mathbf{k}') \tag{30}$$

Then for the matrix $[C(E)]_{\nu\nu}$ we yield

$$[C(E)]_{\nu\nu} = \frac{2\pi}{\hbar}\sum_{\mathbf{k}}\sum_{\mathbf{k}'}\Big\{\frac{16\pi^4\hbar^4}{m^2S^2}\Big(\frac{(k^2-k'^2)^2}{4} - \frac{\pi^2\nu^2}{d^2}\Big)^2|F_f(\mathbf{k}-\mathbf{k}')|^2(k^2-\mathbf{k}\cdot\mathbf{k}')\times$$
$$\times\delta(E-E_{\nu\mathbf{k}})\delta(E-E_{\nu\mathbf{k}}) +$$
$$+\frac{16\pi^4\hbar^4}{m^2S^2}k^2(k^2-k'^2)^2|F_f(\mathbf{k}-\mathbf{k}')|^2\sum_{\mu\neq\nu}\frac{\mu^2\nu^2}{(\nu^2-\mu^2)^2}\delta(E-E_{\nu\mathbf{k}})\delta(E-E_{\mu\mathbf{k}'})\Big\} \tag{31}$$

$$[C(E)]_{\nu\nu'} = \frac{2\pi}{\hbar}\sum_{\mathbf{k}}\sum_{\mathbf{k}'}\Big\{ - \frac{16\pi^4\hbar^4}{m^2S^2}\frac{\nu'^2\nu^2}{(\nu^2-\nu'^2)^2}\big(k^2-k'^2\big)^2\mathbf{k}\cdot\mathbf{k}'|F_f(\mathbf{k}-\mathbf{k}')|^2 \times$$

$$\times\delta(E-E_{\nu\mathbf{k}})\delta(E-E_{\nu'\mathbf{k}'})\Big\}.(32)$$

Due to the quasicontinuity of $\mathbf{k}$, we can replace the summation by integration and obtain for the matrix:

$$[C(E)]_{\nu\nu} = \frac{4\pi^2}{\hbar}\frac{\Delta^2}{d^2}S\xi^2\int_0^{2\pi}\Big\{k_\nu^2(1-\cos\theta)\big(\frac{\pi^2\nu^2}{d^2}\big)^2F(\xi k_{\nu\nu}) + \quad (33)$$

$$+k_\nu^2\sum_{\mu\neq\nu}\frac{\mu^2\nu^2}{(\nu^2-\mu^2)^2}(k_\mu^2-k_\nu^2)^2F(\xi k_{\mu\nu})\Big\}d\theta \quad (34)$$

$$[C(E)]_{\nu\nu'} = -\frac{4\pi^2}{\hbar}\frac{\Delta^2}{d^2}S\xi^2\int_0^{2\pi}k_\nu k_{\nu'}\cos\theta\frac{\nu'^2\nu^2}{(\nu^2-\nu'^2)^2}(k_\nu^2-k_{\nu'}^2)^2F(\xi k_{\nu\nu'})d\theta \quad (35)$$

in the designations of the 1st paragraph. Note that from the definition k_ν it concludes that

$$E_\nu + \frac{\hbar^2k_\nu^2}{2m} = E_{\nu'} + \frac{\hbar^2k_{\nu'}^2}{2m}, \quad (36)$$

where E_ν is determined by the expression (24). As an outcome we arrive exactly to the result of the "first" approach.
We may conclude that violation of Eq. (14) in using of the "first" approach is insignificant.

2 Contribution of different types of distortions of the surface in resistivity of thin conductive films

Let us consider a conductive film, whose surface distortion is defined by the two functions $f_A(\boldsymbol{\rho})$ and $f_B(\boldsymbol{\rho})$ for $z = \frac{1}{2}d$ and $z = -\frac{1}{2}d$, respectively.
The aim is to study the influence on the conductivity of 2 types of distortions - "symmetric" ($f_A = -f_B$), when the film is thickened at once in both sides (keeping the center still), and "antisymmetric" ($f_A = f_B$), when the film deflects as a whole, not changing the thickness. For the simplicity we consider a system with 2 filled levels ($N = 2$) and use the "first" method.
The perturbation has the following form:

$$U = -V\big[f_A(\boldsymbol{\rho})\delta(z-\frac{1}{2}d) - f_B(\boldsymbol{\rho})\delta(z+\frac{1}{2}d)\big] \quad (37)$$

In order to apply the Eq. (5), it is necessary to find $\langle\nu\mathbf{k}|U|\nu'\mathbf{k}'\rangle$, that falls into 2 terms:

$$\langle\nu\mathbf{k}|U|\nu'\mathbf{k}'\rangle = A + B,$$

$$A = \langle\nu\mathbf{k}| - Vf_A(\boldsymbol{\rho})\delta(z - \frac{1}{2}d)|\nu'\mathbf{k}'\rangle = -\frac{V}{S}\phi_\nu(\frac{1}{2}d)\phi_{\nu'}(\frac{1}{2}d)\int e^{i\boldsymbol{\rho}\cdot(\mathbf{k}'-\mathbf{k})}f_A(\boldsymbol{\rho})d\boldsymbol{\rho}, \quad (38)$$

$$B = \langle\nu\mathbf{k}|Vf_B(\boldsymbol{\rho})\delta(z + \frac{1}{2}d)|\nu'\mathbf{k}'\rangle = \frac{V}{S}\phi_\nu(-\frac{1}{2}d)\phi_{\nu'}(-\frac{1}{2}d)\int e^{i\boldsymbol{\rho}\cdot(\mathbf{k}'-\mathbf{k})}f_B(\boldsymbol{\rho})d\boldsymbol{\rho}, \quad (39)$$

$$|A|^2 = \frac{A_\nu A_{\nu'}}{S}\int e^{i\boldsymbol{\rho}\cdot(\mathbf{k}'-\mathbf{k})}\langle f_A(\boldsymbol{\rho}')f_A(\boldsymbol{\rho}'+\boldsymbol{\rho})\rangle d\boldsymbol{\rho}, \quad (40)$$

$$|B|^2 = \frac{A_\nu A_{\nu'}}{S}\int e^{i\boldsymbol{\rho}\cdot(\mathbf{k}'-\mathbf{k})}\langle f_B(\boldsymbol{\rho}')f_B(\boldsymbol{\rho}'+\boldsymbol{\rho})\rangle d\boldsymbol{\rho}, \quad (41)$$

where $A_\nu = V\phi_\nu^2(\frac{1}{2}d) = V\phi_\nu^2(-\frac{1}{2}d)$, and

$$A^*B = -\frac{1}{S^2}D_\nu D_{\nu'}I(\mathbf{k}'-\mathbf{k}), \quad (42)$$

where $D_\nu = V\phi_\nu(\frac{1}{2}d)\phi_\nu(-\frac{1}{2}d)$ (not forgetting that ϕ is a real number) and cross integral

$$I(\mathbf{k}'-\mathbf{k}) = \int e^{i(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)\cdot(\mathbf{k}'-\mathbf{k})}f_A(\boldsymbol{\rho}_1)f_B(\boldsymbol{\rho}_2)d\boldsymbol{\rho}_1\boldsymbol{\rho}_2. \quad (43)$$

Then depending on the evenness/oddness of the states ϕ_ν the coefficients D_ν will coincide with A_ν (within accuracy of the sign). Particularly, it is known that the state $\nu = 1$ is even, and $\nu = 2$ is odd.

The two cases of "symmetric" and "antisymmetric" are now considered. Let us take $f_A = -f_B$. When we construct the quantity

$$|\langle\nu\mathbf{k}|U|\nu'\mathbf{k}'\rangle|^2 = |A|^2 + |B|^2 + 2Re(A^*B) \quad (44)$$

and yield

$$|\langle 1\mathbf{k}|U|1\mathbf{k}'\rangle|^2 = \frac{A_1^2}{S}\cdot 4\Delta^2\xi^2F(\xi|\mathbf{k}-\mathbf{k}'|) \quad (45)$$

$$|\langle 1\mathbf{k}|U|2\mathbf{k}'\rangle|^2 = |\langle 2\mathbf{k}|U|1\mathbf{k}'\rangle|^2 = 0 \quad (46)$$

$$|\langle 2\mathbf{k}|U|2\mathbf{k}'\rangle|^2 = \frac{A_2^2}{S}\cdot 4\Delta^2\xi^2F(\xi|\mathbf{k}-\mathbf{k}'|). \quad (47)$$

The conductivity is

$$\sigma = \frac{e^2\hbar^3}{8\pi m^2F(0)d\Delta^2\xi^2}\Big(\frac{k_1^2}{A_1^2} + \frac{k_2^2}{A_2^2}\Big), \quad (48)$$

conditioned by transitions of carriers from 1st to 1st and from 2nd to 2nd levels ν, while the transitions $1 \to 2$ and $2 \to 1$ are forbidden. For the case $f_A = f_B$ by performing similar calculations, we obtain

$$\sigma = \frac{e^2\hbar^3}{8\pi m^2 F(0) d\Delta^2\xi^2}\Big(\frac{k_1^2}{A_1A_2} + \frac{k_2^2}{A_1A_2}\Big), \tag{49}$$

that is conditioned by the transitions $1 \to 2$ and $2 \to 1$.

We may conclude the following: the deflection of the film as a whole ("antisymmetric" case) contributes to the conductivity via the transitions of carriers between the levels of different evenness, while the "symmetric" distortion gives the contribution via transitions between the levels of same evennes.

Finally the formula (analogous to Eq. (11)) for the general case of different distortions f_A and f_B:

$$[C(E)]_{\nu\nu'} = \delta_{\nu,\nu'}\frac{2m^2S}{\pi\hbar^5}k_\nu^2 A_\nu\Big[\sum_{\mu\leftrightarrow\nu} A_\mu\Delta_s^2\xi_s^2F_s(0) + \sum_{\mu\not\leftrightarrow\nu} A_\mu\Delta_a^2\xi_a^2F_a(0)\Big], \tag{50}$$

where quantities with indeces s and a correspond to above defined terms, responsible for other f:

$$f_s = \frac{f_A - f_B}{2} \tag{51}$$

$$f_a = \frac{f_A + f_B}{2}, \tag{52}$$

and the levels of equal and different evenness are denoted by signs "$\leftrightarrow$" and "$\not\leftrightarrow$".

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